

Exercise 81

Find h' in terms of f' and g' .

$$h(x) = f(g(\sin 4x))$$

Solution

Calculate $h'(x)$ by using the chain rule repeatedly.

$$\begin{aligned} h'(x) &= \frac{d}{dx}[f(g(\sin 4x))] \\ &= f'(g(\sin 4x)) \cdot \frac{d}{dx}[g(\sin 4x)] \\ &= f'(g(\sin 4x)) \cdot g'(\sin 4x) \cdot \frac{d}{dx}(\sin 4x) \\ &= f'(g(\sin 4x)) \cdot g'(\sin 4x) \cdot (\cos 4x) \cdot \frac{d}{dx}(4x) \\ &= f'(g(\sin 4x)) \cdot g'(\sin 4x) \cdot (\cos 4x) \cdot (4) \\ &= 4f'(g(\sin 4x))g'(\sin 4x) \cos 4x \end{aligned}$$